The Laslett Tune Shift for the B Factory*

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Abstract

A straightforward evaluation of the Laslett tune shift formula yields a substantial value of 0.08 for the LER. However, only a very small part ($\simeq 4\times10^{-4}$) of this is a tune *spread* due to the direct space-charge forces from the beam particles. The rest is a bona fide tune *shift* due to the induced charges and currents. The tune spread is negligible in comparison with the beam-beam tune spread. The tune shift can be compensated for by an appropriate choice of working point. However, it should be remembered that this tune shift is intensity-dependent, as is the case for other transverse impedance effects. Therefore the nominal setting for the working point should track the current as it decreases due to particle attrition during the collider's normal operation.

1 Introduction.

For proton synchrotrons one of the main factors which determines the choice of the injection energy is the value of $\Delta\nu$, the linear Laslett tune shift [1]. Proton accelerators have operated with linear tune shifts up to 0.7, but it is generally considered prudent to design the accelerator so that $\Delta\nu < 0.25$. On the other hand, in present electron storage rings the linear Laslett tune shift is completely negligible and is generally ignored. However, as we show here, the B factory's LER has $\Delta\nu \simeq 0.08$ which is larger than the beam-beam tune shift. What is this Laslett tune shift? Should we worry about it, and how does its effect differ between electron and proton rings?

2 The space-charge tune shift.

For the sake of this discussion, we consider only the vertical tune shift of the LER. For a tri-gaussian bunch the Laslett tune shift is given by[1]

$$\Delta\nu_y = -\frac{(N/B)\beta_y r_e}{2\pi\beta^2\gamma^3\sigma_y(\sigma_x+\sigma_y)} - \frac{N\beta_y r_e}{\pi\gamma}\left\{\frac{1}{\beta^2\gamma^2 B}\frac{\epsilon_1}{h^2}+\frac{\epsilon_1}{h^2}+\frac{\epsilon_2}{g^2}\right\} \tag{1}$$

where:

- N = total number of particles in the ring;
- β_y = vertical beta-function;
- r_e = classical radius of the particle;
- $\beta,\ \gamma$ are the usual relativistic quantities;

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[1]A ring-average of the right hand side of Eq. (1) is implicit.

- $\sigma_y,\ \sigma_x$ are the vertical and horizontal rms beam sizes;
- $B = \sqrt{2\pi}\sigma_z/s_b$ is the bunching factor (σ_z = rms bunch length, s_b = bunch spacing);
- h = half-height of the vacuum chamber;
- g = half-height of the magnetic gap;
- ϵ_1 and ϵ_2 are image form factors for the vacuum chamber and magnet gap, discussed below.

The first term in Eq. (1) is the direct space charge term, originally derived by Kerst [2] for a uniform cross section round beam and by Teng [3] for a uniform cross section elliptical beam. For a beam with a gaussian distribution cross section the oscillations are linear only for amplitudes much smaller than the rms beam size. Therefore this first term applies only to a particle near the center of the bunch. Particles that oscillate with larger amplitude experience, on the average, smaller transverse forces and have, therefore, smaller tune shifts. Similarly, particles that are longitudinally displaced from the bunch center experience smaller tune shifts because of the smaller local charge density. Therefore, like the beam-beam tune shift, the direct space charge tune shift is a measure of the tune *spread* in the beam. In addition, the direct space charge tune shift depends upon the peak longitudinal bunch density, so for low-energy rings (a property of many proton synchrotrons) the space charge tune shift limits the maximum stable beam current at the injection energy.

It is interesting to speculate why proton rings are able to operate with much larger tune spreads, due to the space charge term, than colliding rings are able to obtain for beam-beam tune spreads. It is probably because the space charge tune spreads are more uniform around the ring and the main effect is to spread the particles tune across the resonances caused by the magnetic guide fields of the ring rather than to excite new resonances. The beam-beam tune spread is much more localized in azimuth and it excites many new resonances. This hypothesis is somewhat justified, since it is well known that careful attention to removing the resonances caused by the magnetic elements in proton storage rings can increase the allowable linear Laslett tune shift. Actually much of this discussion is academic to us since we will find that the direct space charge term for the LER is negligible anyway.

The remaining terms in Eq. (1) are due to induced charges and currents in the vacuum chamber walls and the magnet pole faces. It should be pointed out that the effects of cavities and other discontinuities in the vacuum chamber are not included in Eq. (1) and we have assumed that the vacuum walls are uniformly smooth around the ring. The discontinuities are taken care of by an effective transverse impedance $Z_\perp$, and have been discussed in detail in the design report [4]. In present day electron storage rings the cavity and other ring impedances dominate the tune shift.

Generally the forces produced by the induced charges and currents are fairly linear in the region where the beam normally lies, provided it does not get too close to the vacuum chamber walls. This is because the size of the vacuum chamber is generally much larger than the size of the beam. Therefore the tune shift produced by the induced charges and currents is approximately the same for all particles, so this does not translate into a tune spread. A more quantitative discussion, in terms of the relevant skin depth, is provided in the numerical application below.

The first term in the curly brackets in Eq. (1) is due to the induced electric charge and current in the vacuum chamber wall. This term depends upon the peak current ($\propto 1/B$), but again the effects of the electric and magnetic fields nearly cancel (to order $1/\gamma^2$) so it is small. This leaves us with the last two terms, which appear because the finite resistivity of the vacuum chamber wall allows the low frequency component of the current and magnetic field to diffuse through the wall. The second term in the curly bracket is the effect of the induced charges left on the vacuum chamber wall, which are no longer canceled by the induced current, which has diffused through the wall. The last term is due to the induced current at the magnetic pole faces, where the magnetic field lines terminate (we assume infinite permeability).

3 Application to the SLAC/LBL/LLNL B factory.

As mentioned above, a ring average of Eq. (1) is implicit. For our present purposes we take this averaging into account by simply replacing $\sigma_x,\ \sigma_y$ and β_y by their ring-averages $\bar{\sigma}_x,\ \bar{\sigma}_y$ and $\bar{\beta}_y$. From the B factory conceptual design report [4] we take or derive the following parameters:

Table 1: LER parameters (APIARY 6.3–D design).

$\bar{\beta}_x,\ \bar{\beta}_y$ [m]	18.7, 15.5
$\bar{\sigma}_x,\ \bar{\sigma}_y$ [mm]	1.31, 0.236
σ_z [cm]	1.0
s_b [m]	1.26
N	9.8×10^{13}
E [GeV]	3.1
$h,\ g$ [cm]	2.5, 3.5

For parallel-wall vacuum chambers (width≫height) and parallel magnet poles the image coefficients are [1]

$$\epsilon_1 = \frac{\pi^2}{48}, \qquad \epsilon_2 = f_m \frac{\pi^2}{24} \tag{2}$$

where f_m is the fraction of the ring occupied with bending magnets, $f_m = \rho/R$, where ρ is the magnetic bending radius and R the average ring radius. For the LER $f_m = 0.0871$, and therefore

$$\epsilon_1 = 0.2056, \qquad \epsilon_2 = 0.03582 \tag{3}$$

The true values for ϵ_1 and ϵ_2 for the LER are probably smaller than these because, of course, the B factory is not designed with parallel-plate vacuum chamber nor parallel magnet poles; however, the above values should give a rough, if pessimistic, estimate of the effect.

From Table 1 we calculate $\gamma = 6,067$ and $B = 2.0 \times 10^{-2}$. The classical electron radius is $r_e = 2.818 \times 10^{-15}$ m. Eq. (1) then yields

$$\begin{aligned} \Delta\nu_y &= -4.17 \times 10^{-4} - \left\{1.00 \times 10^{-7} + 7.39 \times 10^{-2} + 6.56 \times 10^{-3}\right\} \\ &= -0.08 \end{aligned} \tag{4}$$

where each numerical value in the first line corresponds exactly to each term in Eq. (1).

4 Discussion.

As mentioned earlier, the induced current diffuses through the vacuum chamber wall due to the finite conductivity of the metal. The skin depth is given by

$$\delta(\omega) = \frac{c}{\sqrt{2\pi\sigma\omega}} \tag{5}$$

where σ is the conductivity and ω is the relevant angular frequency. Because there is a gap in the bunch train, the lowest frequency in the Fourier spectrum of the beam is the frequency with which the gap comes around, namely the revolution frequency, $f_0 = 136$ kHz. The bunch frequency, $f_b = 238$ MHz, is much higher than this. We assume the conductivity to be that of copper, $\sigma = 5.38 \times 10^{17}$ s^{-1}. The skin depth at the revolution frequency ($\omega_0 = 2\pi f_0$), is $\delta_0 = 177$ μm. At higher frequencies the skin depth is, of course, smaller than this. Since δ_0 is much smaller than the vacuum chamber height, or even the wall thickness, we conclude that the nonlinear effects from the induced current are negligible. Thus the gap in the beam train has only a negligible effect on the induced current that has diffused through the wall, and therefore we conclude that all bunches experience essentially the same tune shift due to the forces from induced currents and charges.

Since the skin depth at the bunch frequency is $\ll 177$ μm, we conclude, *a fortiori*, that all particles in a given bunch also experience the same linear forces.

5 Conclusions.

Since the forces from induced currents and charges have essentially only linear effects, they cause the same tune shift on all particles, and therefore these forces do not lead to a detrimental tune spread. This intensity-dependent tune shift, estimated here at 0.08, can be compensated by a shift in the working point that tracks the current as it decreases due to normal particle attrition.

The direct space-charge forces do cause a tune spread, which we have estimated here at 4×10^{-4}, but it is much smaller than the beam-beam tune spread. Therefore this space-charge effect is not important in practice.

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